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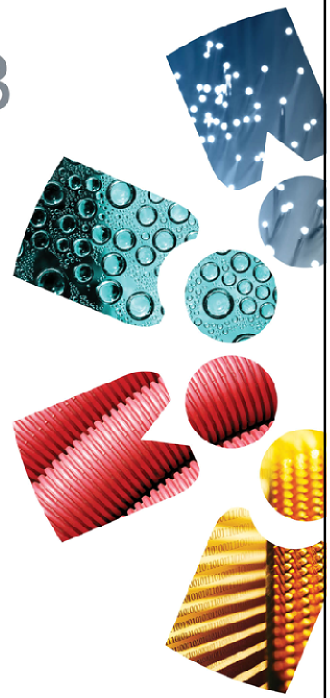

# Breakthroughs in Back Orders

Matt Drake, Ph.D., CFPIM
Duquesne University, Pittsburgh, PA

Co-Authors: David Pentico and Carl Toews

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## Presentation Overview

- EOQ with partial backordering
- Issues with non-linearly-changing backorder rate models
- Approximating the non-linearly-changing rates with constant and linear models
- Experimental results
- Conclusion

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# Rationale for Partial Backordering

- While some customers are willing to wait for delivery, others are not
  - Order cancellations
  - Supplier fulfills order using expensive methods of alternative supply
- Partial backordering EOQ model
  - Fraction (β) of demand that cannot be filled from stock is backordered
  - Remaining fraction (1-β) of demand is lost

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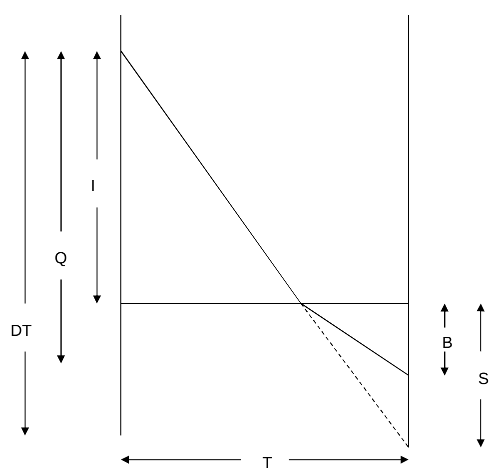
# Basic EOQ with Partial Backordering

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# Basic EOQ Results

- Objective: Minimize average cost per period

$$\Gamma(T,F) = \frac{C_o}{T} + \frac{C_h D T F^2}{2} + \frac{\beta C_b D T (1-F)^2}{2} + C_l D(1-\beta)(1-F)$$

- First-order conditions yield optimal decisions
  - $T^* = \sqrt{\frac{2C_o}{DC_h}\left[\frac{C_h+\beta C_b}{\beta C_b}\right] - \frac{[(1-\beta)C_l]^2}{\beta C_h C_b}}$
  - $F^*(T^*) = \frac{(1-\beta)C_l + \beta C_b T^*}{T^*(C_h + \beta C_b)}$
- Feasibility condition:

$$\beta \geq \beta^* = 1 - \frac{\sqrt{2C_o C_h D}}{C_l D}$$

Otherwise, use either basic EOQ with no backordering or don't stock the item and lose all sales! (Choose the cheaper option.)

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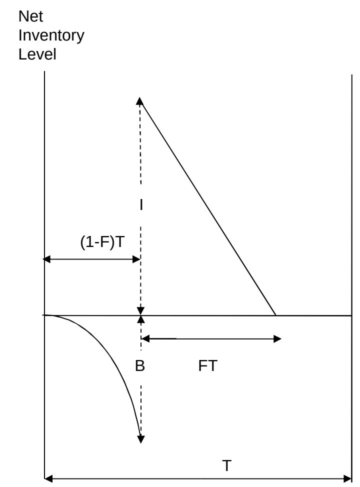
# EOQ with Linearly-Increasing Backorder Rate

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## Time-Dependent Backorder Rates

- Linearly-increasing backorder rate (Toews et al. 2011)
  - $\beta(t) = \beta_0 + (1 - \beta_0)\left(\frac{t}{(1-F)T}\right)$ for $0 \leq t \leq (1 - F)T$
- Exponential backorder rate (San Jose et al. 2006)
  - $B(\tau) = \rho\exp(-\alpha\tau)$ for $\tau > 0$
- Rational backorder rate (San Jose et al. 2005)
  - $B(\tau) = \rho/(1 + \alpha\tau)$ for $\tau > 0$

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## Issues with Non-Linearly-Changing Backorder Rates

- No closed-form solution like the constant and linearly-changing rate models have
- Solution procedures
  - Non-linear programming
  - Iterative process involving a search procedure such as Newton's Method
- Time consuming and harder to automate
- Difficult for many (perhaps *most*) managers to understand

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## Main Research Question

- Is it worth the hassle to use a non-linearly-changing backorder rate model to manage inventory, or will a constant or linearly changing backorder rate model perform well enough in practice?

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## Approximating Solutions for the Non-Linearly-Changing Backordering Rates

- Estimating the constant backorder rate or the starting point for the linear rate requires an estimate of the stockout interval for the non-linear rate model
- Three possibilities based on iterations discussed in paper:
  1. Use only the first estimate, skipping re-estimation
  2. Use alternative with lowest estimated cost
  3. Use alternative with lowest actual cost *(DIFFICULT!)*

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# Experimental design

- Five parameters of interest
  - Demand per period [20, 200]
  - Fixed ordering cost [0.5, 5, 50]
  - Backorder cost per period [0.5, 5.0]
  - Ratio of cost of lost sale to backorder cost per period [2, 5]
  - Backorder resistance [0.10, 0.25, 0.50, 1.00]
- 2 * 3 * 2 * 2 * 4 = 96 problem instances

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# Results for Exponential Model

| | | Constant-Exponential | | Linear-Exponential | |
|---|---|---|---|---|---|
| | | Avg | Max | Avg | Max |
| a = .10 | Alt 1 | 1.00391 | 1.0120 | 1.00370 | 1.0121 |
| | Alt 2 | 1.00405 | 1.0120 | 1.00384 | 1.0121 |
| | Alt 3 | 1.00372 | 1.0113 | 1.00354 | 1.0114 |
| a = .25 | Alt 1 | 1.01389 | 1.0421 | 1.01342 | 1.0429 |
| | Alt 2 | 1.01522 | 1.0421 | 1.01500 | 1.0429 |
| | Alt 3 | 1.01168 | 1.0347 | 1.01152 | 1.0352 |
| a = .50 | Alt 1 | 1.03062 | 1.0859 | 1.03011 | 1.0970 |
| | Alt 2 | 1.04037 | 1.0859 | 1.03966 | 1.0970 |
| | Alt 3 | 1.01838 | 1.0473 | 1.01948 | 1.0575 |
| a = 1.0 | Alt 1 | 1.06562 | 1.1587 | 1.06683 | 1.1938 |
| | Alt 2 | 1.12622 | 1.2846 | 1.16452 | 1.4592 |
| | Alt 3 | 1.01608 | 1.0505 | 1.02126 | 1.0885 |

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# Results for Rational Model

| | | Constant-Rational | | Linear-Rational | |
|---|---|---|---|---|---|
| | | Avg | Max | Avg | Max |
| a = .10 | Alt 1 | 1.00329 | 1.0119 | 1.00354 | 1.0120 |
| | Alt 2 | 1.00345 | 1.0119 | 1.00368 | 1.0120 |
| | Alt 3 | 1.00327 | 1.0116 | 1.00339 | 1.0114 |
| a = .25 | Alt 1 | 1.01129 | 1.0416 | 1.01195 | 1.0423 |
| | Alt 2 | 1.01291 | 1.0416 | 1.01345 | 1.0423 |
| | Alt 3 | 1.01062 | 1.0361 | 1.01029 | 1.0349 |
| a = .50 | Alt 1 | 1.02442 | 1.0847 | 1.02730 | 1.0893 |
| | Alt 2 | 1.03385 | 1.0847 | 1.03612 | 1.0893 |
| | Alt 3 | 1.01831 | 1.0623 | 1.01788 | 1.0558 |
| a = 1.0 | Alt 1 | 1.05307 | 1.1489 | 1.05187 | 1.1672 |
| | Alt 2 | 1.07626 | 1.2812 | 1.13427 | 1.3107 |
| | Alt 3 | 1.01893 | 1.0467 | 1.01909 | 1.0525 |

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# Summary of the Results

- Approximation performance is best for small values of *a*
- Alternative 3 (selecting estimate with lowest actual cost) performs significantly better for high values of *a*
- Cost ratios were slightly lower for:
  - Demand of 20 compared to 200
  - Higher ordering costs (50 compared to 5 and 0.5)
  - Backorder cost of 0.5 compared to 5.0
  - Lost sales to backorder cost ratio of 2 compared to 5

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## Conclusions

- The functional form of the backorder rate does not make much of a difference
- Exotic functional forms often do little more than muddy the analytical waters
- Researchers examining other factors such as deterioration or inflation can reasonably assume a constant backorder rate
- Identifying simpler forms for modeling the additional considerations should be a goal because this increases the practical applicability of the model

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## Survey

www.tinyurl.com/lc3s3fm

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